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Nicholas St. John Green, and from Professor William Ripley Nichols, acknowledging their election into the Academy.

It was voted to grant the use of the Hall of the Academy on the morning of Thursday, December 12, to the Massachusetts Historical Society.

The President called the attention of the Academy to a volume richly illustrated, entitled "Revision of the Echini," by Alexander Agassiz.

Alexander Agassiz then exhibited specimens of the Albertype and Woodburytype methods of photo-engraving employed in illustrating his work.

The Recording Secretary exhibited a new form of Theodolite-Magnetometer.

A letter was then read from Dr. Robert Amory, describing some specimens of objects exposed to the fire of November 9, which led to a discussion in which most of the members present participated.

Six hundred and fifty-second Meeting.

January 14, 1873. — MONTHLY MEETING.

The CORRESPONDING SECRETARY in the chair.

The Corresponding Secretary read a letter from Professor Döllen, of Pulkowa, acknowledging his election, as Foreign Honorary Member, into the Academy.

Professor J. M. Peirce read the following paper by Professor J. M. Rice of Annapolis, and Professor W. W. Johnson of St. John's College, Maryland, "On a New Method of obtaining the Differentials of Functions, with especial reference to the Newtonian Conception of Rates or Velocities."

The fundamental conceptions which have been employed in the development of the elementary theorems of the Differential Calculus are four in number, namely, that of *Infinitesimal Differences*, that of *Limits*, that of *Derived Functions*, and that of *Rates* or *Fluxions*.

While the first and more recently the second have received greater attention and more systematic development, the last, the Conception of Rates, has an important advantage in the readiness with which its definition is apprehended in consequence of the familiarity of the phenomena of motion.*

The most important objections which have been made to the "Method of Fluxions," as developed by Newton and his followers, are those directed against the methods employed in deducing the fluxions of the different functions. These are usually geometrical methods, often indirect and wanting in generality, even when founded upon well-known and satisfactorily demonstrated properties. The algebraic methods, also, which are employed, are frequently dependent upon an objectionable use of infinite series.

While a *constant rate* is easily measured by the increment received in a unit of time, a difficulty is encountered when an attempt is made to employ increments in the measurement of a *variable rate*. This difficulty probably gave rise to the common method, in which a comparison of rates is effected by the conception of simultaneous infinitesimal increments; to these, while divested of magnitude, ratios are ascribed which are really the ratios of the rates of quantities simultaneously varying.

The method of limits is another device for obtaining the values of the same ratios.

This last expedient, having been adopted by Maclaurin (perhaps the ablest writer on Fluxions), the impression has become prevalent that recourse to it affords the only satisfactory method of treating the subject of rates.

The following is an attempt to supply a direct method of proving the elementary theorems of the Differential Calculus, which is independent of all consideration of limits, of infinitesimals, and of algebraic series.

DEFINITIONS AND NOTATION.

When a quantity varies uniformly, the constant numerical measure of its rate is the increment received in the unit of time. When, however, the variation is not uniform, we would define the numerical measure of the rate at any instant as the increment which would be received in a unit of time, if the rate remained uniform from and after the given instant.

This definition corresponds with the usage of mechanics, in accordance with which a body moving with a variable velocity is said to have at a given instant a velocity which would carry it thirty-two feet in one second.

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* See Art. 42, p. 72, *Traité élémentaire de la Théorie des Fonctions et du Calcul Infinitésimal.* Par COURNOT. Paris, 1841.

To avoid departing too much from well-established usage, the term *differential* will be frequently used in this paper instead of *rate.*

The *rate* or *differential* of x will be denoted by Dx, and that of f, x by $D(f, x)$.

The rate of the independent variable, or the value of Dx, is regarded as arbitrary in the same sense that the value of x is arbitrary.

Thus, particular values of these two quantities may constitute the data of a question like the following: What is the value of $D(x^2)$, when x has the value 10 and Dx the value 4?

To differentiate a function of x is to express $D(f, x)$ in terms of x and Dx in such a manner as to furnish a general formula by which $D(f, x)$ may be computed for any given values of x and of Dx.

ELEMENTARY PROPOSITIONS.

The following propositions are immediate deductions from the above method of measuring rates: —

I. *The Differential of* $x + h$.

Since any simultaneous increments of x and of $x + h$ must be identical, the increments which would be received by each, if they continued to vary uniformly with the rates denoted by Dx and $D(x + h)$, are equal. Hence the rates are equal, or

$$D(x + h) = Dx.$$

II. *The Differential of* $x + y$.

Since any increment of $x + y$ is the sum of the simultaneous increments of x and of y, the same relation exists between the increments which would be received if x and y (and consequently $x + y$) continued to vary uniformly with the rates denoted by Dx, Dy, and $D(x + y)$. Hence

$$D(x + y) = Dx + Dy.$$

III. *The Differential of* mx.

Since any increment of mx must be m times the corresponding increment of x, the same relation must exist between the increments which would be received if x (and consequently mx), continued to vary uniformly with the rates denoted by $D(x)$ and $D(mx)$. Therefore

$$D(mx) = m\,Dx.$$

THE RATIO OF THE RATES OF A VARIABLE AND ITS FUNCTION.

Let y denote a linear function of x such that

$$y = mx + b. \qquad [1]$$

By propositions I. and III.

$$Dy = m\,Dx,$$

or

$$\frac{Dy}{Dx} = m. \qquad [2]$$

In this case, the ratio of the rate, or differential, of the function to that of the independent variable is constant, its value being independent not only of x, but also of Dx. Thus, if we give to Dx any arbitrary value, it is evident from equation [2], that Dy must take a corresponding value such that the ratio of these quantities shall always retain the constant value m.

Assuming rectangular co-ordinate axes, if y be made the ordinate corresponding to x as an abscissa, the point (x, y) will, as x varies, generate a straight line. The direction of the motion of the point is constant, and depends upon the value of m. Since $\frac{Dy}{Dx}$ is equal to m, it is the trigonometrical tangent of the constant inclination of the direction of the generating point to the axis of x.

When y is not a linear function of x, the direction of the motion of the generating point is variable, and consequently the value of $\frac{Dy}{Dx}$ is variable.

Making, now, the arbitrary quantity Dx a constant, Dy will be a variable. Suppose, then, that, the generatrix having arrived at a given point, the ordinate y continues to vary uniformly with the rate denoted by Dy at the given point; the value of $\frac{Dy}{Dx}$ will become constant. The generatrix will now continue to move uniformly in the direction of the curve at the given point, and therefore the value which $\frac{Dy}{Dx}$ has at this point is that of the trigonometrical tangent of the inclination of the curve to the axis of x at this point. The line *now* described by the generatrix is called a tangent line to the curve, in accordance with the following general definition: The tangent line to a curve at a given point is the line passing through the point, and having the direction of the curve at that point.

IV. *The Ratio of the Rates is independent of their Absolute Values.*

Since the direction of the curve (or of the tangent line) at the point having a given abscissa is determined by the form of the function, or equation, to the curve, the value of $\frac{Dy}{Dx}$, which is the trigonometrical tangent to the inclination of this direction, must be independent of the arbitrary quantity Dx, which merely determines the velocity of the generating point.

In general, the value of $\frac{D(f,x)}{Dx}$ will change with that of x; $\frac{D(f,x)}{Dx}$ is, therefore, independent of Dx, but is generally a function of x.

$D(f,x)$, when expressed in terms of x and of Dx, is of the form

$$D(f,x) = \phi x \,.\, Dx$$

in which ϕx is another function of x.

In the ordinary methods, the introduction of an equivalent proposition is, for the most part, avoided by rejecting from the ultimate value of $\Delta(f,x)$ all terms containing powers of Δx higher than the first.

We shall now proceed to deduce, from the four elementary propositions hitherto proved, the differentials of the functions both algebraic and transcendental. These propositions are here recapitulated for convenience of reference: —

I. $D(x+h) = Dx$.

II. $D(x+y) = Dx + Dy$.

III. $D(mx) = m\,Dx$.

IV. $\frac{D(f,x)}{Dx}$ is independent of Dx, but is generally a function of x.

THE DIFFERENTIAL OF THE SQUARE.

Let $z = x + h$, then $Dz = Dx$, [1]

and $z^2 = x^2 + 2hx + h^2$, and $D(z^2) = D(x^2) + 2h\,Dx$. [2]

Dividing [2] by [1], $\frac{D(z^2)}{Dz} = \frac{D(x^2)}{Dx} + 2h$, and since $h = z - x$,

$$\frac{D(z^2)}{Dz} - 2z = \frac{D(x^2)}{Dx} - 2x. \qquad [3]$$

Since $\frac{D(z^2)}{Dz}$ is a function of z only, and $\frac{D(x^2)}{Dx}$ a function of x only, while z and x are any two values of the independent variable (h being arbitrary), the functional expression which constitutes either member of equation [3] does not change its value with the independent variable, hence

$$\frac{D(x^2)}{Dx} - 2x = c \text{ (a constant)}, \qquad [4]$$

or

$$D(x^2) = 2x\,Dx + c\,Dx. \qquad [5]$$

To determine the unknown constant c, we differentiate, by equation [5], the identity

$$(mx)^2 = m^2 x^2,$$

obtaining

$$2mx \cdot m\,Dx + cm\,Dx = m^2\,2x\,Dx + m^2 c\,Dx,$$

or

$$cm(1-m)\,Dx = 0.$$

$$\therefore \qquad c = 0,$$

since m and Dx have arbitrary values,

$$\therefore \qquad D(x^2) = 2x\,Dx. \qquad [a]$$

Equation [a] may also be deduced from [3] by the following method:—

In equation [3] not only are x and z entirely independent, since h is arbitrary, but Dx and Dz are no longer restricted, for $\frac{D(z^2)}{Dz}$ will not change its value if we suppose Dz to have any value greater or less than Dx. We may therefore put

$$z = mx.$$

Introducing this value in [3], we obtain

$$\frac{D(m^2 x^2)}{D(mx)} - 2mx = \frac{D(x^2)}{Dx} - 2x,$$

or

$$m\frac{D(x^2)}{Dx} - 2mx = \frac{D(x^2)}{Dx} - 2x;$$

$$\therefore \qquad (m-1)\frac{D(x^2)}{Dx} = (m-1)\,2x.$$

Whence, since m is arbitrary,

$$\frac{D(x^2)}{Dx} = 2x.$$

The process used in the case of those functions whose differentials it is desirable to deduce independently is, in each instance, similar to that used above in the case of $D\,(x^2)$, and may be thus described: —

We assume a new variable z, connected with x by a relation admitting of a comparison of Dz and Dx, and at the same time such, that $D\,(f, z)$ and $D\,(f, x)$ may likewise be compared; in other words, such that the relation between z and x, and also between f, z and f, x, can be differentiated without the introduction of unknown differentials, except those denoted by $D\,(f, x)$ and $D\,(f, z)$.

By division, the ratios $\frac{D\,(f, x)}{Dx}$ and $\frac{D\,(f, z)}{Dz}$ are introduced in a single equation. The arbitrary constant introduced in the assumed relation between z and x is then eliminated, and the equation reduced to such a form that one member is apparently a function of z, and the other of x. This last process we call the *separation of the variables.*

As x and z may denote any two values of the independent variable, the apparent functions mentioned above will necessarily be identical in form, and (since they constitute the two members of an equation) identical also in value. This value will be constant, since either member of the equation is a functional expression, which does not change its value with x.

The determination of this constant is then effected by the differentiation of some algebraic identity.

The Differential of the Product.

From the above expression for $D\,(x^2)$, we obtain the Differential of the Product, thus: —

$$(x+y)^2 = x^2 + 2\,xy + y^2.$$

$$\therefore \quad 2\,(x+y)\,(Dx+Dy) = 2\,x\,Dx + 2\,D\,(xy) + 2\,y\,Dy,$$

$$\text{or} \quad x\,Dx + y\,\mathrm{D}x + x\,Dy + y\,Dy = x\,Dx + D\,(xy) + y\,Dy,$$

$$\therefore \quad D\,(xy) = y\,Dx + x\,Dy.^* \qquad [\mathrm{b}]$$

From this result the *Differential of the Quotient* is easily obtained.

The Differential of the Power.

Let $z = rx$, then $Dz = r\,Dx$, [1]

and $z^m = r^m x^m$, then $D\,(z^m) = r^m\,D\,(x^m)$, [2]

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* This method of deriving $D\,(xy)$ from (Dx^2) is taken from *Vince's Fluxions.*

Dividing [2] by [1], $\frac{D(z^m)}{Dz} = r^m \frac{D(x^m)}{r\,Dx}$,

and, since $r = \frac{z}{x}$, $\frac{D(z^m)}{Dz} = \frac{z^{m-1}}{x^{m-1}} \frac{D(x^m)}{Dx}$.

Separating variables, by dividing by z^{m-1},

$$\frac{1}{z^{m-1}} \frac{D(z^m)}{Dz} = \frac{1}{x^{m-1}} \frac{D(x^m)}{Dx}. \qquad [3]$$

By a train of reasoning precisely similar to that employed in obtaining equation [4] of the article on $D(x^2)$

we prove $$\frac{1}{x^{m-1}} \frac{D(x^m)}{Dx} = C_m \qquad [4]$$

C_m being used to denote the constant, because it may be a function of m.

From [4] $$D(x^m) = C_m\, x^{m-1}\, Dx. \qquad [1]$$

By similar notation $D(x^n) = C_n\, x^{n-1}\, Dx$,

and $$D(x^{m+n}) = C_{m+n}\, x^{m+n-1}\, Dx.$$

To determine C_m.

If $m = 1$ in equation [1] $Dx^1 = C_1\, x^0\, Dx$

$$\therefore \qquad C_1 = 1. \qquad [2]$$

Differentiating the identity

$$x^{m+n} = x^m\, x^n,$$

$$C_{m+n}\, x^{m+n-1}\, Dx = x^n\, C_m\, x^{m-1}\, Dx + x^m\, C_n\, x^{n-1}\, Dx,$$

and dividing by $x^{m+n-1}\, Dx$,

$$C_{m+n} = C_m + C_n, \qquad [3]$$

making $m = n$ $\qquad C_{2n} = 2\ C_n$,

making $m = 2n$ $\qquad C_{3n} = 3\ C_n$, &c.

$\therefore$ $\qquad C_{pn} = p\ C_n$; (p being a positive integer). [4]

Making $n = 1$ in [4], $\qquad C_p = p\ C_1 = p$. [5]

Again, in [4], putting $n = \frac{q}{p}$, q and p being positive integers,

$$p\, C_{\frac{q}{p}} = C_q = q\,;$$

whence $$C_{\frac{q}{p}} = \frac{q}{p}. \qquad [6]$$

Again, in [3], making $m = o$,

$$C_o = o,$$

and, making $m = -n$, $\quad C_n = -C_n,$

or, making $n = \frac{p}{q}$, $$C_{\frac{p}{q}} = -\frac{p}{q}. \qquad [7]$$

From [6] and [7] we have generally

$$C_m = m,$$

and substituting this value of C_m in [1],

$$D\,(x^m) = m\, x^{m-1}\, D\, x. \qquad [c]$$

THE DIFFERENTIAL OF THE LOGARITHMIC FUNCTION.

Let $\quad z = m\,x, \qquad$ then will $\quad D\,z = m\,D\,x, \qquad [1]$

and $\quad \log z = \log m + \log x$, and $D\,(\log z) = D\,(\log x). \qquad [2]$

Dividing [2] by [1],

$$\frac{D\,(\log z)}{D\,z} = \frac{D\,(\log x)}{m\,.\,D\,x} = \frac{x}{z}\,.\,\frac{D\,(\log.\,x)}{D\,x},$$

since $\frac{1}{m} = \frac{x}{z}$, $\therefore\; z\,\frac{D\,(\log z)}{D\,z} = x\,\frac{D\,(\log x)}{D\,x} = B. \qquad [3]$

B denotes a constant depending upon the base of the system of logarithms. Denoting by b this base, and by $\log_b$ a corresponding logarithm, we have

$$D\,(\log_b x) = \frac{B \cdot D\,x}{x}, \qquad [4]$$

and by similar notation $D\,(\log_a x) = \frac{A \cdot D\,x}{x}$.

A relation between A and B is found by differentiating the identical

equation $\quad \log_a x = \log_a b\, \log_b x,$

thus obtaining $\quad \frac{A\,D\,x}{x} = \log._a b \cdot \frac{B\,D\,x}{x},$

whence $\quad A = B \cdot \log_a b = \log_a b^B;$

$\therefore \quad a^A = b^B.$

The form of this equation shows that the value of the expression b^B is independent of the value of b; it is, therefore, a numerical constant, and may be denoted by e.

i. e. $$b^B = e;$$

$\therefore$ $$B \log_e b = 1,$$

whence $$B = \frac{1}{\log_e b}.$$

Introducing this value of B in equation [4], we obtain

$$D(\log_b x) = \frac{Dx}{\log_e b.\, x}.$$

If $b = e$, we have $$D(\log_e x) = \frac{Dx}{x}. \qquad [d]$$

e is known as the Napierian base. The computation of its approximate numerical value is deferred until after the introduction of Maclaurin's Theorem.

The *logarithmic differentials* of the Power, Product and Quotient may be deduced by means of the above result, in the usual way, since the demonstration is dependent on the four elementary propositions only.

The Differential of the Exponential Function.

Let $$z = x + h, \text{ then } Dz = Dx, \qquad [1]$$

also $$a^z = a^{x+h} = a^h \cdot a^x \; [3] \text{ and } D(a^z) = a^h\, D(a^x). \qquad [2]$$

Dividing equation [2] by the product of [1] and [3], we obtain the desired form

$$\frac{1}{a^z} \cdot \frac{D(a^z)}{Dz} = \frac{1}{a^x} \cdot \frac{D(a^x)}{Dx} = c.$$

Whence $$D(a^x) = c\, a^x\, Dx. \qquad [4]$$

To determine c, we differentiate the identity

$$x = \log_a a^x,$$

thus $$Dx = \frac{D(a^x)}{\log_e a \cdot a^x} = \frac{c\, a^x\, Dx}{\log_e a \cdot a^x};$$

$\therefore$ $$c = \log_e a.$$

Substituting this value of c in [4], we obtain

$$D\,(a^x) = \log_e a \cdot a^x \cdot D\,x. \qquad [e]$$

THE DIFFERENTIALS OF THE TRIGONOMETRICAL FUNCTIONS.

Let $z = x + h,$ then $D\,z = D\,x,$ [1]

also $\sin z = \sin x \cos h + \cos x \sin h,$

$$= \cos h \cdot \sin x + \sin h \sqrt{1 - \sin^2 x}.$$

$$D\,(\sin z) = \cos h \cdot D\,(\sin x) + \sin h \; \frac{-2 \sin x\, D\,(\sin x)}{2\sqrt{1 - \sin^2 x}}$$

$$= \left[\cos h - \sin h \cdot \frac{\sin x}{\cos x}\right] D\,(\sin x). \qquad [2]$$

Combining [1] and [2]

$$\frac{D\,(\sin z)}{D\,z} = \frac{D\,(\sin x)}{D\,x} \cdot \frac{\cos h \cos x - \sin h \sin x}{\cos x}$$

$$= \frac{\cos z}{\cos x} \cdot \frac{D\,(\sin x)}{D\,x}, \qquad [3]$$

or, separating variables,

$$\frac{1}{\cos z} \cdot \frac{D\,(\sin z)}{D\,z} = \frac{1}{\cos x} \cdot \frac{D\,(\sin x)}{D\,x} = c\,;$$

$$\therefore \quad D\,(\sin x) = c \cdot \cos x \cdot D\,x. \qquad [4]$$

To determine c (x being the circular measure of the angle)

Put $\cos x = \sin\left(\frac{\pi}{2} - x\right).$

By [4] $D\,(\cos x) = c \cdot \cos\left(\frac{\pi}{2} - x\right) D\left(\frac{\pi}{2} - x\right),$

or $D\,(\cos x) = -\,c \sin x\, D\,x.$ [5]

From [4] and [5] $D\,(\tan x) = c \sec^2 x\, D\,x.$ [6]

Now in [4] c cannot be greater than unity, for if it were, $D\,(\sin x)$ would exceed $D\,x$ for all values of x less than a certain value. Hence, x and $\sin x$ starting together from zero, $\sin x$ would, for these values, exceed x, which is impossible.

Again, from [6] c cannot be less than unity, for then $D\,(\tan x)$

would be less than Dx for values of x below a certain fixed value, and x and $\tan x$, starting together from zero, $\tan x$ would be less than x for these values, which again is impossible;

$$\therefore \qquad C = 1.$$

Introducing this value of c in [4] [5] and [6] we obtain

$$D(\sin x) = \cos x \, Dx. \qquad [f]$$

$$D(\cos x) = -\sin x \, Dx. \qquad [g]$$

$$D(\tan x) = \sec^2 x \, Dx. \qquad [h]$$

The President communicated the following letter: —

CORDOBA, November 7, 1872.

TO THE PRESIDENT OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES.

DEAR SIR, — Upon my departure from home on the undertaking in which I am at present engaged, the Academy had the goodness to aid my plans materially by appropriating the sum of $ 500 in gold from the Rumford Fund for the purpose of supplying me with a star spectroscope and astronomical photometer, — with permission to transfer them at the same price to the Argentine government for permanent use in this hemisphere, in case that the means for their purchase by the National Observatory should become available.

The instruments were ordered without delay, and are now in my possession, in good order. But the numerous delays arising from the German war of 1870 postponed both the construction and the transportation of the apparatus; and the instruments did not reach Cordoba until all my energies had been so severely tasked in other directions as to preclude me from undertaking any spectroscopic or photometric observations for the present.

I had hoped to offer to the Academy some results obtained with these instruments, in recognition of the valuable aid so opportunely and generously afforded; and I do not yet relinquish the hope of so doing at some future time. Meanwhile I have the satisfaction of being able to announce that the funds have been provided for the acquisition of these instruments by the observatory, and have the honor of transmitting them herewith to the Academy, with my cordial thanks.

It may have some interest for the Academy to know that, in the two years which have elapsed since my arrival in Cordoba, the observatory buildings have been erected and equipped, where before that time was a desolate and waste expanse of broken ground, almost untrodden, except by prairie-dogs, foxes, and iguanas. A thorough survey of the entire southern heavens has been made, comprising a determination of the positions and brightness of all stars to a limit below the seventh magnitude inclusive, and the whole work then repeated for the twofold purpose of detecting errors and of recognizing any important changes in the stars themselves. This revision will in all probability have been completed before the close of 1872. Standards of magnitude have for the first time been established for each tenth of a unit, as far as the eighth magnitude, throughout the circumference of the heavens, and selected in that portion of the Northern Hemisphere which has an equal meridian altitude for this observatory and for the average of northern ones. A thorough revision of the constellations of this hemisphere has been accomplished, and definite boundaries established, which, if accepted by astronomers, as I have reason to believe will be the case, will put an end to the confusion that has hitherto existed. The zone observations for a Southern Catalogue have been organized and are now going on systematically through the whole region between the Tropic of Capricorn and the eightieth degree of South Declination, the positions of nearly seven thousand stars having been already determined. Three campaigns for longitude determinations by telegraph have been carried out with the view of improving the map of the continent; and when the pending determination of the longitude between this observatory and the National Observatory of Chile shall have been completed, the positions of many points in South America will be known with a precision quite comparable to that with which the principal points in the United States have been established.

Furthermore, a system of meteorological observations is now organizing, as a national establishment, intended to embrace all portions of this Republic from Bolivia to the Straits of Magellan, and from the Andes to the Pacific. Funds have been voted for furnishing the needful instruments to all who are able and willing to carry on the observations, and the whole organization, although under my charge for the present, is made independent of the observatory. Thus there is reason to hope that the strange and hitherto almost unknown meteorological peculiarities of this singular region will within a few years become well understood.

I yet indulge the hope that it may be within my power, in some future communication, to offer to the American Academy some results of permanent value obtained by means of the instruments which could not have been supplied but through its generous aid. Only the intense application required for the zone-observations delays at present those upon the physical character of the stars whose positions we are determining.

With great respect, I am, dear sir,
Your obedient servant,
B. A. GOULD.

The Treasurer stated that he had received $573, (the value in currency of $500 in gold,) returned by Dr. Gould to the Academy for reasons fully explained in his letter.

Professor J. M. Crafts described a series of tests recently made at the Institute of Technology, on the solubility of sulphide of arsenic in free chlorohydric acid, and on the fire-proof qualities of a concrete.

Professor Wolcott Gibbs announced that he had found that the higher alkaloids, strychnin, brucin, etc., form, with the phosphates and arsenates of uranium and of the metals of the magnesian group, well-defined salts corresponding to the well-known ammonio-magnesic phosphate and arsenate. Many of these salts yield splendid crystals, which promise to extend our power of separating the different alkaloids from each other. The uranic salts are remarkably insoluble, and will probably yield quantitative methods of analysis.

Dr. J. B. S. Jackson exhibited two photographs of the presumed fossil human skeleton lately exhumed at Mentone. He also exhibited a specimen of a urinary calculus, cut in two and polished to show its internal structure.

Six hundred and fifty-third Meeting.

January 29, 1873. — STATED MEETING.

The PRESIDENT in the chair.

The Corresponding Secretary stated that the list of Resident Fellows exceeded in number that allowed by the charter, and